Leibniz Universität Hannover

Mathematic and Physic (Admission year 2009)

Exam: Part A1 „ Mathematic “

All answers are to be reasoned!

Allowed materials:

- Non-programable calculator without Text-input
- Writing-pad, with a white paper identified by name and register number, for your draft

Exam Time: Part A1 and A2 together 40 Minutes

4 Questions (Part A1)

Name:…………………………… Register Nr. : ……………………….

Hints:

- Write on all pages containing answers, your name and register number.
- All printed question pages should be given back completely.

By corrector only:

Question Nr.	Point(Mark)	Corrector	Exam Director
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Mathematics

Time Allowed —20 *Minutes*

There are *FOUR* questions on the paper.
ALL questions will be used for assessment.

Question 1. Show that the matrix

$$A=\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix},$$

satisfies the equation $A^2 - 4A - 5I = 0$. Hence, determine A^{-1}.

Question 2. A rectangle with sides parallel to the coordinate axes is inscribed in the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Find the largest possible area for this rectangle.

Question 3. Solve the initial-value problem

$$y'' + 2y' + 2y = 0, \quad y(0) = 2, \ y'(0) = -3.$$

Question 4. Determine whether the vector field

$$\mathbf{F} = (xy - \sin z)\mathbf{i} + \left(\frac{1}{2}x^2 - \frac{e^y}{z}\right)\mathbf{j} + \left(\frac{e^y}{z^2} - x\cos z\right)\mathbf{k},$$

is conservative in $D = \{(x, y, z): z \neq 0\}$, and find a potential if it is.

Leibniz Universität Hannover

Mathematic and Physic (Admission year 2009)

Exam: Part A2 „ Physic “

All answers are to be reasoned!

<u>Allowed materials:</u>

- Non-programable calculator without Text-input
- Writing-pad, with a white paper identified by name and register number, for your draft

Exam Time: Part A1 and A2 together 40 Minutes

4 Questions (Part A2)

Name:…………………………………………………… Register Nr. : ……………………….

Hints:

- Write on all pages containing answers, your name and register number.
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By corrector only:

Question Nr.	Point(Mark)	Corrector	Exam Director
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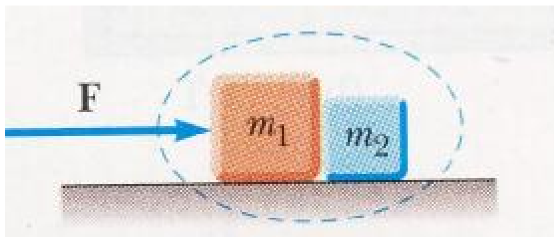
PHYSICS

1) Two blocks of masses "**m1=3kg**" and "**m2=1kg**" are placed in contact with each other on a frictionless horizontal surface. A constant horizontal force "**F=10N**" is applied to the block of mass **m1**. Determine the magnitude of the contact force between the two blocks.



2) If you cover the top half of a lens, what happens to the appearance of the image of the object?

3) A spherical balloon contains a positively charged object at its center. As the balloon is inflated to a greater volume while the charged object remains at the center, does the electric potential at the surface of the balloon increases, decreases, or remain the same? How about the magnitude of the electric field?

4) A positive point charge "**q**" of mass "**m**" is released from rest in a uniform electric field "**E**" directed along the horizontal. Derive an equation for the kinetic energy of the charge after it has moved a distance "**x**".

Electric Circuits and
Principles of Electrical Engineering (Admission year 2009)

Exam: Part B „ Electric circuits and Principles of Electrical Engineering “

All answers are to be reasoned!

Allowed materials:

- Non-programable calculator without Text-input
- Writing-pad, with a white paper identified by name and register number, for your draft

Exam Time : part B (40 Minutes)

4 Questions (Part B)

Name:………………………………………………… Register Nr. : ……………………….

Hints:

- Write on all pages containing answers, your name and register number.
- All printed question pages should be given back completely.

By corrector only:

Question Nr.	Point(Mark)	Corrector	Exam Director
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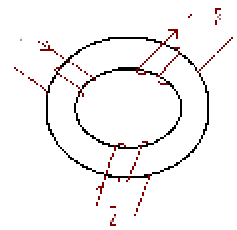
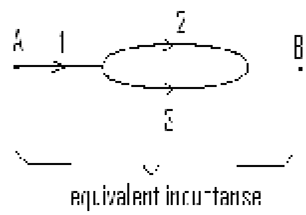
Electric Circuits and Principles of Electrical Engineering

1- Self inductance for each coil in the following circuit is L=2 Henry and absolute mutual inductance M= 1 Henry.

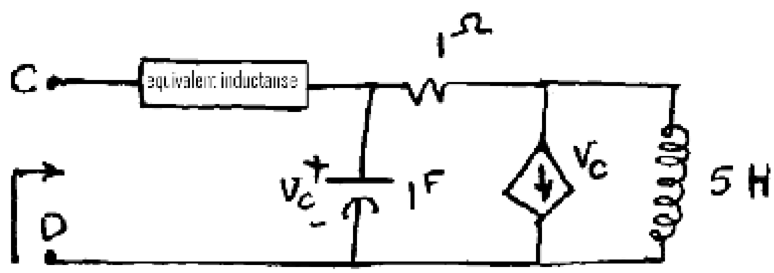


The coils are connected As:



A. Calculate the equivalent inductance between points A and B

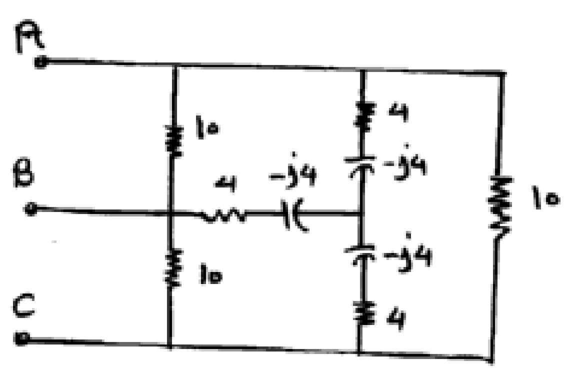
B. The equivalent inductance is used in a circuit as below. Calculate the impedance of circuit seeing from points C and D.



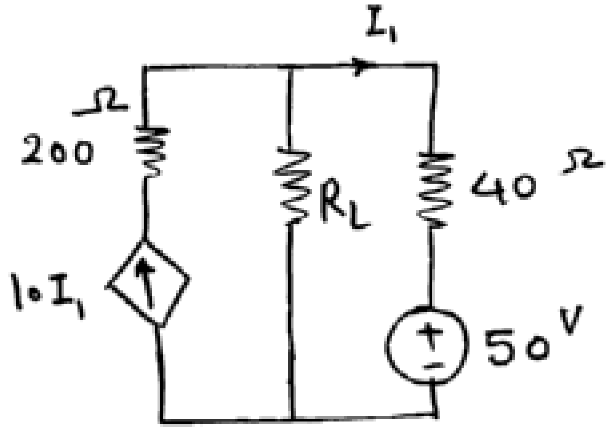
2- The circuit depicted below is fed with an AC three phase source with phase to phase voltage of 380 volt.

A. Calculate three phase line currents. (all impedances are in Ω)

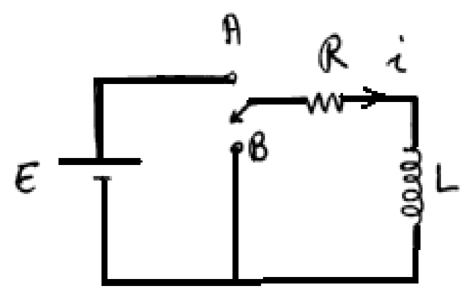
B. If two watt-meters used at lines A and B for power measuring, what are the values shown by each watt-meter.

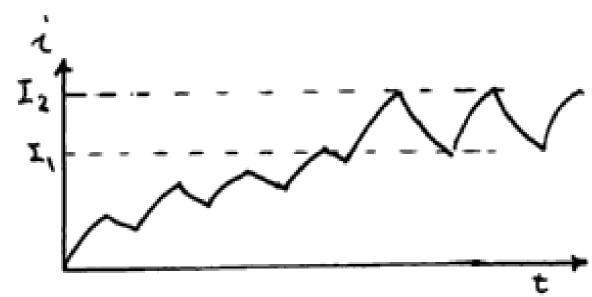


3- In the following Circuit Calculate R_L such that It absorbs maximum power.



4- In circuit shown below the switch is toggled between A and B periodically with T=L/R in second.
After several switching the wave shape of current is stabled and changing between two levels of I_1 and I_2. Calculate current levels of I_1 and I_2.





Signals / Systems and
Linear Systems and Control (Admission year 2009)

Exam: Part C1 „ Signals/Systems “

All answers are to be reasoned!

<u>Allowed materials:</u>

- Non-programable calculator without Text-input
- Writing-pad, with a white paper identified by name and register number, for your draft

Exam Time: Part C1 and Part c2 together 40 minutes

4 Questions (Part C1)

Name:……………………………………………….. Register Nr. : ……………………….

Hints:

- Write on all pages containing answers, your name and register number.
- All printed question pages should be given back completely.

By corrector only:

Question Nr.	Point(Mark)	Corrector	Exam Director
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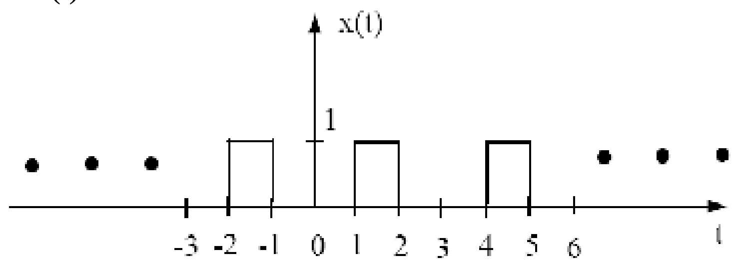
Signals and systems

Question 1: Consider the continuous-time LTI system described by the input-output relationship:

$$y(t) = \int_{t-1}^{t+1} x(t)\,dt$$

1.1. Is this system stable? (Demonstrate your answer).
1.2. Determine and plot the impulse response $h(t)$ of this system.
1.3. Consider that the input to this system is the periodic signal $x(t)$ below with period T=3. Determine and plot several periods of the output, $y(t)$ of the system to the periodic signal $x(t)$.



Question 2: Consider a right-sided sequence $x[n]$ with z-transform:

$$X(z) = \frac{1}{(1-\frac{1}{2}z^{-1})(1-z^{-1})}$$

2.1 Plot pole-zero and region of convergence (ROC) for this sequence in z plane.
2.2 Carry out a partial fraction expansion of $X(z)$; that means find constants A and B such that:

$$X(z) = \frac{A}{(1-\frac{1}{2}z^{-1})} + \frac{B}{(1-z^{-1})}$$

2.3 Find $x[n]$ using properties of z-transform.

Question 3: Consider the continuous-time LTI system with impulse response:

$$h(t) = \frac{\sin[5(t-2)]}{p(t-2)}$$

3.1. Plot the magnitude $|H(jw)|$ and phase $\angle H(jw)$ (two separate plots) of the frequency response of this system. Show as much detail as possible.
3.2. If the input $x(t) = \cos(2t + p)$ is given to the above system, determine a simple closed form expression for the Fourier transform of the input.
3.3. Determine a simple, closed-form expression for the time-domain output of this system to the input.

Question 4: Consider an even sequence $x[n]$ (i.e., $x[n] = x[-n]$) with rational z-transform $X(z)$. From the definition of the z-transform show that:

$$X(z) = X(\frac{1}{z})$$

Signals / Systems and
Linear Systems and Control (Admission year 2009)

Exam: Part C2 „ Linear Systems and Control “

All answers are to be reasoned!

Allowed materials:
- Non-programable calculator without Text-input
- Writing-pad, with a white paper identified by name and register number, for your draft

Exam Time: Part C1 and Part c2 together 40 minutes

4 Questions (Part C2)

Name:……………………………………………….…… Register Nr. : ……………………….

Hints:

- Write on all pages containing answers, your name and register number.
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Control I

Problem 1:

A control system is modeled by the following differential equation:

$$\ddot{y}(t) + a\,\dot{y}(t) + y(t) = 1(t)$$

In which $1(t)$ is unit step function defined as:

$$1(t) = \begin{cases} 1 & t\ \mathbf{f}\ 0 \\ 0 & t\ \mathbf{p}\ 0 \end{cases}$$

a) Find the transfer function $G(s) = \dfrac{y(s)}{1(s)}$
b) For what values of a the system becomes stable?
c) Plot the output of the system ($y(t)$) when a equals zero.

Problem 2:

The process transfer function in a control system is as follows:

$$G(s) = \frac{y(s)}{u(s)} = \frac{k(s-2)}{s+1}$$

In which k is a positive parameter.

a) Plot the open loop $G(jw)$ in the complex plane for $k = 1$.
b) By plotting the Nyquist diagram of $G(jw)$, find the number of unstable closed poles of the system when k varies from zero to infinity (unity negative feedback being used for the system).

Control II

Problem 1:

Consider a system defined by the following transfer function ($y(t)$ is the output and $u(t)$ is the input):

$$G(s) = \frac{y(s)}{u(s)} = \frac{s+4}{s+3}$$

a) Plot the root locus of the closed loop system for a pure gain controller $G_c(s) = -k$ for positive gains k

b) Plot the root locus of the closed loop system for a dynamic controller $G_c(s) = -\frac{k}{s}$ for positive k.

c) Explain what is the purpose of inserting such a controller for this system?

Problem 2:

The state space model of a control system is as follows: ($u(t)$ is the input and $y(t)$ is the output)

$$\begin{cases} \dot{x}(t) = Ax(t) + bu(t) \\ y(t) = C^T x(t) \end{cases} ; A = \begin{bmatrix} 0 & 1 \\ 0.5 & 0.5 \end{bmatrix}; b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}; C^T = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

a) Is the system stable?

b) Is the system completely controllable?

c) Is the system completely observable?

d) Is it possible to find a state feedback gain $u(t) = [k_1 \quad k_2]x(t)$, such that the closed loop poles are assigned at $l_1 = -2, l_2 = -4$? If the answer is positive find k_1 and k_2. If not explain why?