

Fault Tolerant Fuzzy Control Allocation for Overactuated Systems

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Abstract— An adaptive fault tolerant control systems are vital in many industrial systems. Redundancy is a practical approach to decrease the effects of faults in systems. Redundancy in actuators can also increase system reliability and flexibility. This paper proposes a fuzzy control allocation method that can allocate control signal among actuators to increase reliability and maneuverability in healthy conditions and tolerating faults in faulty conditions. Using fuzzy logic is an intelligent way to adaptively change the gains of control allocation in different operating conditions.

Keywords— *Fault Tolerant Control; Control Allocation; Fuzzy Logic; Overactuated Systems.*

I. INTRODUCTION

Actuator redundancy has been used for design of fault tolerant control systems. This redundancy yields multiple ways to implement control signal derived by the controllers. The freedom in allocating control signal among actuators can increase system reliability, flexibility and maneuverability. This degree of freedom can also be used to design fault tolerate control systems. Control allocation is an algorithm that coordinates different actuators to produce a desired virtual control effort.

The simplest control allocation method is based on unconstrained least squares algorithms and modification of the solution to remain in the limits [1]. Daisy chaining is another method which has been broadly used in flight redundancy management. This method divides the actuators in certain groups and sequentially uses them when needed [2]. Direct Allocation is based on pseudo inverse but the solution of this method lies in the constraints [3,4]. This method is condensed into a constrained optimization problem [5]. Also, there are methods based on linear programming and quadratic programming techniques for error minimization [5-10].

Fault tolerant control is an active research area. Two main fault tolerant methodologies are called active and passive methods [11, 12]. In passive methods, the controller is designed based on possible system changes, which is a robust fault tolerant control [13]. Active methods can be categorized in three groups. The first group is control allocation method that manages the control signal through the redundant

actuators in a way that the effect of fault decreases as much as possible [14-16]. The second group is multiple model approaches that adaptively change or mix models so that the system tolerates fault [17, 18]. The third group is reconfigurable control. This method is based on changing the controller parameters or controller method adaptively based on the error minimization when fault occurs [19-21].

The use of fuzzy logic is motivated for flexible decision making. It is well known that fuzzy logic is much closer to human decision making than the traditional logic. Fuzzy logic proposed by Zadeh is the principle of incompatibility based on a human centered design technique [22].

This paper proposes a fuzzy logic based decision making system to adjust the control allocation gains and use the degree of freedom provided by actuators redundancy to design fault tolerant systems.

The paper is organized as follows. Section II presents the problem. Model of the system, model of fault and fuzzy control allocation are introduced in this section. This is followed by section III, that shows an example to illustrate the fuzzy control allocation. Simulations and discussions is brought in this section. At the end, the section IV contains conclusion and brief discussion about the method.

II. PROBLEM STATEMENT

A. System

Consider the plant described by the following state space equation:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1)$$

Where $x \in R^n$ and $u \in R^m$ are the system states and the control input. The matrix B is column rank deficient:

$$\text{rank}(B) = k < m, \forall x \in R^n \quad (2)$$

The input signal $u(t)$ belongs to a compact set Ω that is defined as:

$$u(t) \in \Omega \equiv \{u \in R^m \mid u^- \leq u \leq u^+\} \quad (3)$$

Where the constraints are defined so that $u^- \equiv [u_1^-, u_2^-, \dots, u_m^-]^T$ and $u^+ \equiv [u_1^+, u_2^+, \dots, u_m^+]^T$ [16].

B. Fault

It will be assumed that the system subject to faults can be written as [14]:

$$\dot{x}(t) = Ax(t) + Bu(t) - BK(t)u(t) \quad (4)$$

and by defining the effectiveness gain matrix $K(t)$ as:

$$K(t) = \text{diag}(k_1(t), \dots, k_m(t)) \quad (5)$$

$$0 \leq k_i(t) \leq 1$$

The state space equation of system facing actuator faults are given as:

$$\dot{x}(t) = Ax(t) + BW(t)u(t) \quad (6)$$

$$W(t) = I - K(t)$$

If $k_i = 0$, then the i th actuator is working perfectly and for $k_i > 0$, the i th actuator is faulty and if $k_i = 1$, then the i th actuator has completely failed.

C. Fuzzy Control Allocation

Control allocation is used to manage the control signal among redundant actuators. In this paper, a control allocation model that was proposed in [15] is used. Fig. 1 shows the model and is generalized for all overactuated systems with control allocation as a manager. Define $\alpha_i, i=1, \dots, m$ as the control allocation gains which should be adjusted. By modeling the control allocation problem in Fig. 1, the control allocation algorithm adjusts the gains. In this paper, a fuzzy logic based gain adjustment for control allocation is proposed.

The control allocation in Fig. 1 is formulated as below:

$$v(t) = \alpha(t)v_0(t) \quad (7)$$

and $\alpha(t)$ is the vector of allocation gains. The aim of Fuzzy logic is to use experts' knowledge to improve the quality of decision makings. It is proposed to use the fuzzy logic in order to adaptively choosing allocation gains in (7).

The expert in a system can define the membership functions and rules of fuzzy logic in that system based on the recognition of the system's behavior. Allocating control signals among the actuators are different in various situations and it differs in every system. In this paper, control allocation

is used to tolerant faults in system, because of this, the expert moreover should have knowledge of system faulty behaviors.

Defining membership functions and rules are based on how to choose the inputs of fuzzy system. The easiest way to choose input is to use an identification algorithm in order to identify faults in each actuator. Defining membership functions and rules are Based on the behavior of each actuator in faulty situations. In this method, the change in one effectiveness gain, just affect the corresponding allocation gain. But it is desired to use all capacities of redundancy i.e. the change in one effectiveness gain should affect all allocation gains.

By adding the output error and derivative of output error as inputs of the system, and designing the membership functions and rules based of them, the control allocation gains compute in a way that the output response of the system tracks the desired value by better quality. So the control allocation can adaptively change the inputs of actuators without changing the controllers and use the complete capacity of redundancy in systems to tolerate faults in system.

III. AN ILLUSTRATIVE EXAMPLE

In this section, a system with redundant actuators is presented. This example illustrates the potential of fuzzy control allocation in a faulty system.

A. System's Model

The method is applied to a linear Boeing 737 model that is a large transport aircraft. It is assumed that the model has no pitch rotation so the lateral dynamic model in an equilibrium point is as [23]:

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$x = [v_b, p_b, r_b, \phi, \psi]^T, u = [\delta_r, \delta_a]^T \quad (7)$$

where v_b, p_b, r_b, ϕ and ψ are lateral velocity (ft/s), roll rate (rad/s), yaw rate (rad/s), roll angle (rad) and yaw angle (rad), respectively. The control inputs are aileron (δ_a) and rudder (δ_r) deflections (deg.). The system matrices A and B are as:

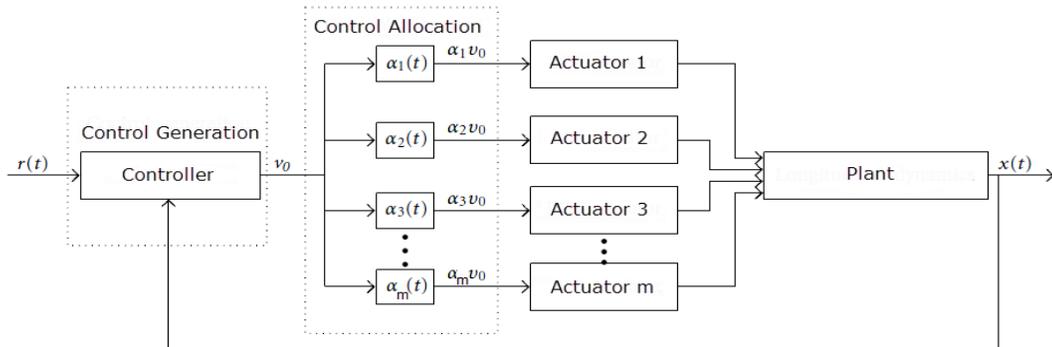


Fig. 1. Control allocation model in a closed loop system compatible with fuzzy logic

$$A = \begin{bmatrix} -0.129 & 28.328 & -774.92 & 32.145 & 0 \\ -0.012 & -1.4419 & 0.9409 & 0 & 0 \\ 0.004 & -0.0409 & -0.1757 & -0.0001 & 0 \\ 0 & 1 & 0.0372 & 0 & 0 \\ 0 & 0 & 1.0007 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0542 & 0.4669 \\ 0.0443 & 0.0200 \\ 0.0025 & -0.0382 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (8)$$

In this example, the yaw angle is considered as the output of the plant:

$$C = [0 \quad 0 \quad 0 \quad 0 \quad 1] \quad (9)$$

B. Parameter Design

The nominal allocation gain vector is as:

$$\alpha^* = [8.6103 \quad -1]^T \quad (10)$$

The nominal allocation gain is chosen such that the first element of $B\alpha^*$ that is the lateral gain for the lateral acceleration is zero [15]. The nominal controller is based on the LQR approach:

$$v_0^*(t) = k_1^{*T} x(t) + k_2^* r(t)$$

$$k_1^* = [-0.8963 \quad 22.1655 \quad -73.6645 \quad 28.8488 \quad 4.0825]^T$$

$$k_2^* = 1 \quad (11)$$

Also the nominal control allocation equation is as:

$$v(t) = \alpha^* v_0^*(t) \quad (12)$$

where $v_0^*(t)$ is the virtual control signal.

C. Simulation Results

The reference input is $r(t)=2.1376$ for $t \geq 0$ which leads to a desired yaw angle of 0.5236 (rad). Consider 80% loss of effectiveness occurs at $t=10$ sec in the aileron actuator. The controller considers to have fixed nominal gains.

Two case of choosing fuzzy system inputs is introduced below:

Case1: just using actuator effectiveness identifiers. So here it is one input for each fuzzy allocation gain.

Case2: adding output error and derivative of output error to actuator effectiveness identifiers. So here it is three inputs for each fuzzy allocation gain and consequently the number of rule bank is huge.

Fig. 2 shows the states of the system in case 1. The yaw angle tracks its reference and the other states have gone to zero, also the effect of fault has been omitted. Fig. 3 shows the allocation

gains that adaptively computed by fuzzy logic. Before fault occurred in rudder actuator, the gains that are computed by fuzzy system are almost similar to the nominal allocation gains and the difference is because of the different knowledge and though of experts. After fault occurred, the aileron allocation gain has not changed because its input has not changed. This is the disadvantage of using case 1.

The input for the fuzzy system is the effectiveness gains of actuators (5) that is identified using identification algorithms. The mamdani rules and triangular membership functions are used as the simplest ones and are defined based on the fact that in this system, the gain of control allocation related to the faulty actuator, increase and the other gain decreases accordingly. Fig. 4 shows the relation between input and output of fuzzy rudder allocation gain and shows it as a function.

Fig. 5 shows the states of the plant in case 2. It is obvious that the yaw angle tracks the reference value and the other states have gone to zero in finite time. Fig. 2 and Fig. 5 are really similar to each other but as can be seen in Fig. 6 and Fig. 3, the allocation gains are different. This may occur because of the use of same controller for these two case. In case 2, two allocation gains can adaptively change and use this redundancy to control the system in a more safety and flexible way.

Fig. 7 shows the surface made by three inputs of fuzzy rudder allocation gain in case 2.

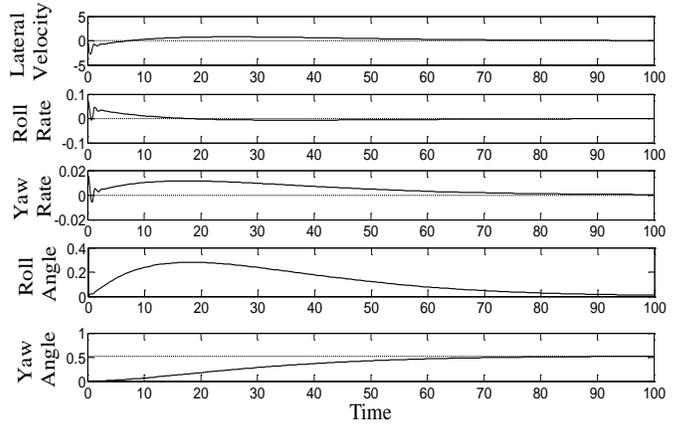


Fig. 2. Plant states (solid line) and the reference states (dashed line) (case1)

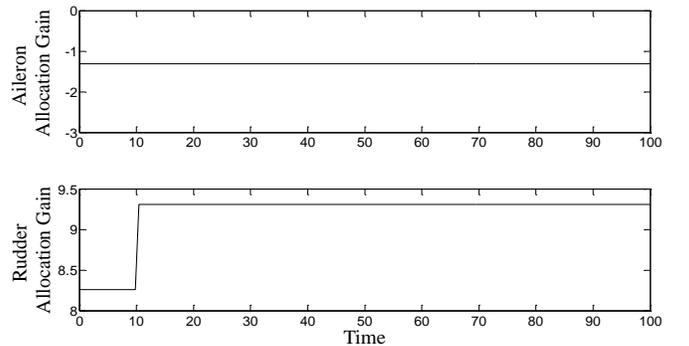


Fig. 3. Control allocation gains computed by fuzzy logic (case1)

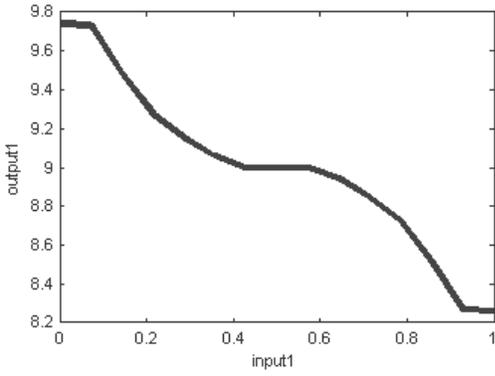


Fig. 4. Fuzzy rudder allocation gain as a function (case1)

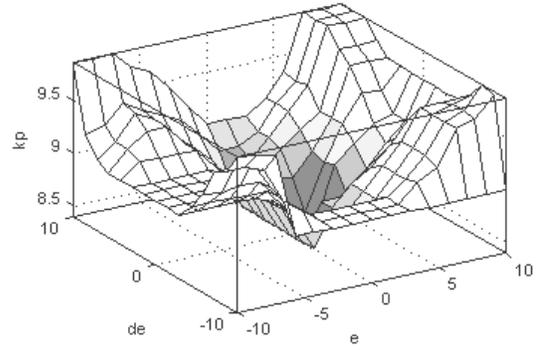


Fig. 7. surface made by three inputs of fuzzy rudder allocation gain (case2)

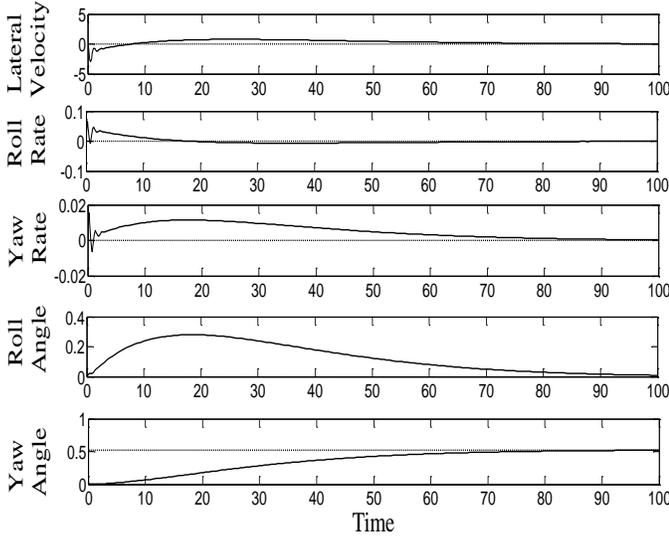


Fig. 5. Plant states (solid line) and the reference states (dashed line) (case2)

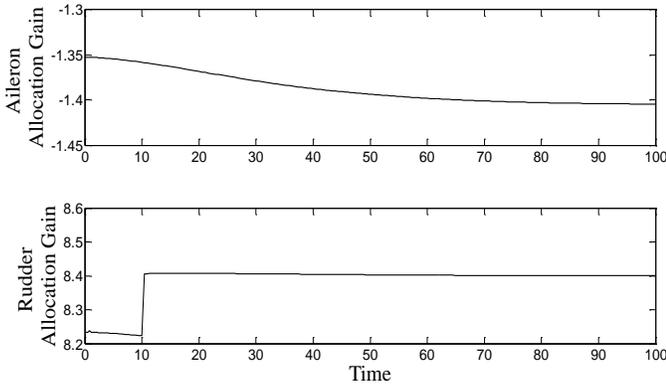


Fig. 6. Control allocation gains computed by fuzzy logic (case2)

IV. CONCLUSION REMARKS

This paper has proposed the fuzzy control allocation method in order to use the capacity of actuator redundancy to decrease the effect of actuator faults. The main property of this method is to use the expert's knowledge to improve the quality of control. Two methods for using inputs are proposed. The case 1 is easier and simpler but the allocation gains can not compensate each other. This problem is solved in case 2 by adding the output error and derivative of output error as inputs of fuzzy logic. Simulation results are used to show the effectiveness of the proposed methodology.

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