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Tuning of Dynamic Matrix Controller for FOPDT Models Using Analysis of Variance

Peyman Bagheri* Ali Khaki-Sedigh**

Advanced Process Automation & Control (APAC) Research Group, K. N. Toosi University of Technology Tehran, Iran
*(e-mail: *bagheri.peyman@ee.kntu.ac.ir, **sedigh@kntu.ac.ir)*

Abstract: Dynamic Matrix Control (DMC) is well known in the MPC family and has been implemented in many industrial processes. In all the MPC methods, tuning of controller parameters is a key step in successful control system performance. An analytical tuning expression for DMC is derived using the analysis of variance (ANOVA) methodology and nonlinear regression. It is assumed that the plants under consideration can be modeled by a First Order plus Dead Time (FOPDT) linear model. This facilitates the derivation of a closed form formulae for the tuning procedure. The proposed method is tested via simulations and experimental work. The plant chosen for practical implementation of the proposed tuning strategy is a nonlinear laboratory scale pH plant. Also, comparison results are provided to show the effectiveness of this method.

Keywords: Dynamic Matrix Control, Analysis of Variance, Nonlinear Regression, Tuning, industrial processes, pH process.

1. INTRODUCTION

Model Predictive Control (MPC) strategies are widely used in industry as Advanced Process Controllers (APC) (Qin and Badgwell, 2003). In the MPC family, Dynamic Matrix Control (DMC) is the most popular in many chemical processes. This popularity is due to the simple structure of the controller. DMC uses step response information and in stable industrial processes, this is easily obtained. DMC, as a model predictive controller, was first proposed in (C. R. Cutler and Ramaker, 1989). As other MPC methods, DMC uses a prediction model. This model is a step response model. The next element in MPC is the objective function (E. F. Camacho and Bordons, 2005). A typical form of the cost function is

$$J=\sum_{i=1}^{P}\big(y(t+i|t)-w(t+i)\big)^2+\sum_{i=0}^{M-1}\lambda\big(\Delta u(t-i)\big)^2 \quad (1)$$

It is desired that the future output values (y) on the considered horizon follow a desired reference trajectory (w) and in the same time, the control effort (Δu) is penalized properly.

The DMC control law is given by

$$\boldsymbol{\Delta\bar{u}}=(\boldsymbol{A}^T\boldsymbol{A}+\lambda\boldsymbol{I})^{-1}A^T\boldsymbol{\bar{e}} \quad (2)$$

Where $\boldsymbol{A}$ is the dynamic matrix, $\boldsymbol{\bar{e}}$ is the vector of predicted errors over prediction horizon P, λ is the move suppression coefficient and $\boldsymbol{\Delta\bar{u}}$ is control effort vector over the control horizon M. In DMC we have a parameter named model horizon N that it affects $\boldsymbol{\bar{e}}$. Finally, sampling time selection is vital for proper DMC performance. Over all, tuning parameters of DMC can be listed as λ, P, M, N and T_s. In this paper we deal with these parameters and try to find the most effective one. In (Wojsznis, 2003), some practical approaches to tuning MPC methods is presented. (Wang, 2003) deals with self adaptive DMC methods. In these works, no formulation for tuning obtained. A new paper that reviews tuning methods for MPC can be found in (Garriga and Soroush, 2010). In (Shridhar and Cooper, 1997) for all parameters of DMC, an equation is obtained based on the FOPDT model approximation of the real plant. Among these tuning parameters, move suppression coefficient is the most effective parameter. The equation for λ in noted paper is according to avoidance of singularity of $(\boldsymbol{A}^T\boldsymbol{A}+\lambda\boldsymbol{I})^{-1}$ and no performance is proposed in this equation. In (Lee, 1994), tuning of MPC is proposed by the mean of robust performance. Also (Iglesias *et al.*, 2006) used the ANOVA to create an analytical equation for λ. In this work a performance index is used to obtain this equation but, there are sever deficiencies associated with the derived formulae. Recently (Neshasteriz *et al.*, 2009) has employed ANOVA for tuning of Generalized Predictive Controller (GPC) for Second Order plus Dead time (SOPDT) models and a new analytical equation for λ is obtained.

In this paper, banks of FOPDT models have been simulated to test the effect of model parameters on the tuning parameter λ. ANOVA is performed on these data to determine the effective parameters contributing to grate changes in tuning parameter. Finally nonlinear regression is employed to obtain a simple but enough accurate tuning equation for λ.

This paper is organized as follows:

In section 2, some deficiencies of previous works lined out. In section 3, a new tuning procedure is described in some details. The next section is tried to show the effectiveness of the proposed tuning equation through simulation test and experimental validation. Finally, conclusions end the paper.

2. TUNING PROCEDURE

In this section, two well known tuning methods are tested via simulation examples. Then, the new tuning procedure is presented in some details.

2.1 Previous tuning methods

Consider the approximated FOPDT model

$$G_m(s) = \frac{Ke^{-\theta s}}{\tau s+1} \tag{3}$$

The tuning equations are presented in Table 1.

Table 1. DMC tuning Methods

Parameters	(Shridhar and Cooper, 1997)	(Iglesias *et al.*, 2006)
T_s	$T_s \le 0.1\tau$ *and* $T_s \le 0.5\theta$	$T_s \le 0.1\tau$ *and* $T_s \le 0.5\theta$
$P = N$	$P = N = \frac{5\tau}{T_s} + k,$ $k = \frac{\theta}{T_s} + 1$	$P = N = \frac{5\tau}{T_s} + k,$ $k = \frac{\theta}{T_s} + 1$
M	Integer, usually from 1 to 6	Integer, usually from 1 to 6
λ	fK^2	$1.631K\left(\frac{\theta}{\tau}\right)^{0.4094}$
f	$\begin{cases} 0 & M = 1 \\ \frac{M}{500}\left(\frac{3.5\tau}{T_s} + 2 - \frac{M-1}{2}\right) & M > 1 \end{cases}$	-

The first test example is process1 in (Shridhar and Cooper, 1997). Figure 1 shows the effect of small N.

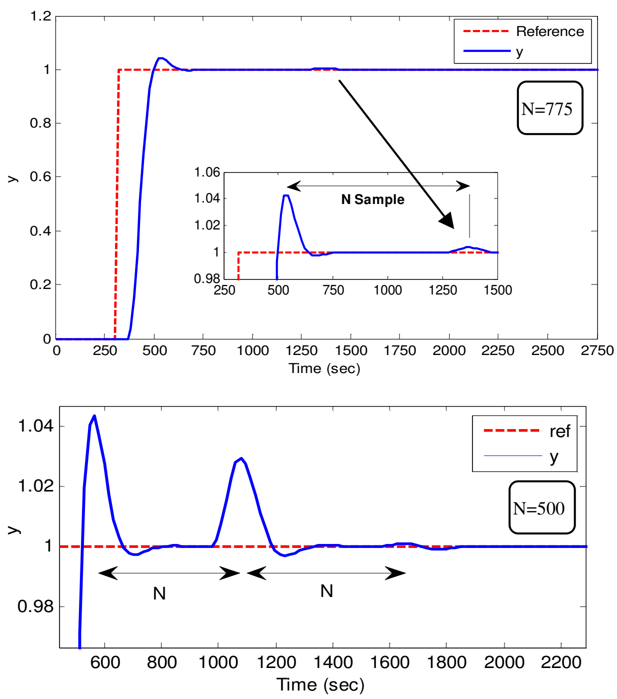

Fig. 1. Effect of small N.

It is shown that every N sample, transient response repeats with damping mood but also in second plot it is shown that if N get smaller this repetitive responses would be undesirable. So if it is possible, it is desirable to choose $N > \frac{5\tau}{T_s} + k$. In (Shridhar and Cooper, 1997), the proposed value for N is not large enough.

Next, consider the equation for λ in table1 (Shridhar and Cooper, 1997). In this equation, performance is neglected and also large overshoot in control signal is encountered. To be precise, in this equation if τ/T_s term get smaller by increasing delay, a larger sampling time and a very small λ is achieved and it yields a fast response that is not desirable for a system with large delay.

The proposed tuning method in (Iglesias *et al.*, 2006) for calculating λ is shown in Table 1. It is noted that:

It is shown in (Shridhar and Cooper, 1997) that $\lambda = fK^2$. Formulation in (Iglesias *et al.*, 2006) shows that, this relation is linear, $\lambda = fK$. With a simple example we can show that if $K < 0.01$ the closed loop responses become so slow.

In the case of small enough delays in comparison with plant time constant, λ will be very small which yields a very fast and maybe unstable response. In (Iglesias *et al.*, 2006), using the analysis of variance it is shown that the parameter Γ (defined later) is not efficient. Reason is that K is used in ANOVA and also the range of Γ is not sufficient. We will show that this parameter is very important and has an effective influence on the closed loop response. To study the (Iglesias *et al.*, 2006) formulation, consider the following FOPDT model

$$G(s) = \frac{0.002e^{-s}}{s+1} \tag{4}$$

According to (Shridhar and Cooper, 1997) formulation, for this system we have $\lambda = 9.28e^{-7}$. And according to (Iglesias *et al.*, 2006) formulation we have $\lambda = 3.3e^{-3}$. Figure 2. Shows the results of these methods, it is shown that (Iglesias *et al.*, 2006) formulation leads to a very slow response.

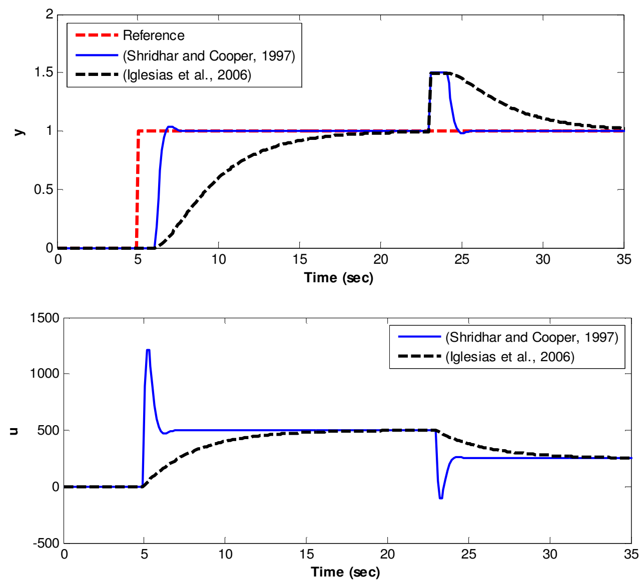

Fig. 2. Closed loop responses.

Also this example shows the response of (Shridhar and Cooper, 1997) method. This method yields to the fast response and 140% overshoot in control signal which is not desirable and executable in practice.

A new method is now presented to overcome the above mentioned problems. The main idea in this paper is using ANOVA and some useful insights to find appropriate nonlinear fitting that is sufficiently accurate and also not complicated. To find an analytical expression for λ, FOPDT model of the plant is used. Note that all the parameters of Table1 except λ and N, adopted in this paper. For λ a new equation will be presented, but N should only be larger than what it is in Table1, empirically $N = 2\frac{5\tau}{T_s} + k$ is a good choice.

2.2. Tuning Procedure

According to (Shridhar and Cooper, 1997), we have $\lambda = fK^2$. So in the following we consider only system delay and time constant of FOPDT and the goal is to find optimal equation for f. Note that in our tuning method we choose $M = 4$ as a good choice, according to Table 1.

Now we construct the model bank, as shown in Table 2, overall we have 7×5^2=175 model. For each model appropriate parameters according to Table 1 are chosen, except for f. Then for each model, f varies from 0.05 to 10 and performance index (5) is calculated. Finally, the optimal value of f is obtained for every model.

$$J = \int_0^\infty \left(r(t) - y(t)\right)^2 dt + \Gamma \int_0^\infty \left(\Delta u(t)\right)^2 dt \tag{5}$$

Table 2. Setup of parameters for analysis of variance

ANOVA parameters	Level Low	Level Low Medium	Level Medium	Level Medium High	Level High
τ	10	40	80	120	160
θ	2	5	15	40	80
Γ	0.1	0.2 0.5	1	2 3	5

After finding the optimal value of f for each model, analysis of variance (Scheffe, 1959) is performed on the optimal tuning parameters as a response vector and model parameters as variables. In these simulations up to 2 way combination between the models parameters are taken into account. Results of ANOVA show that which one of model parameters or a combination of model parameters has more influence on the optimal tuned parameters and also the level of influence is determined by ANOVA. The result of ANOVA analysis is depicted in Table 3. In this table, there are F-values and P-values associated with each parameter and combination of parameters. These values reveal the effect and also the effectiveness level of these parameters on optimal tuned parameters.

Typically there is a cut-off value of 0.05 for P index. That is, any of these sources having a value below the cut-off is considered to be significant. Also, a source with small P value and larger F value has larger influence on optimal tuned parameter. The remaining terms are omitted. According to Table 3 it is shown that, Γ is the most efficient parameter. Effect of θ and τ are countable. To have simple formulation, combinations of parameters are not considered.

Table 3. ANOVA results for DMC

Source	Sum Sq.	d.f.	Mean Sq.	F	Prob>F
τ	0.743	4	0.1858	26.18	5.84e-16
θ	0.577	4	0.1444	20.34	4.71e-13
Γ	316.43	6	52.7383	7431.6	0
$\theta \times \tau$	0.418	16	0.0261	3.68	1.53e-5
$\Gamma \times \tau$	1.706	24	0.0711	10.01	1.29e-19
$\Gamma \times \theta$	1.327	24	0.0553	7.79	1.19e-15
Error	0.93	131	0.0071		
Total	322.13	209			

Now we try to find a meaningful function of these parameters. For this propose, Fig3 is checked in detail. In this figure the optimal value of f is shown in terms of the plant model parameters. Hence,

- A primary formulation for f is in the form of (6)

  $$f = a\Gamma^b \tag{6}$$

- For a fixed time constant, increasing system delay leads to larger f.
- Plots comparisons show that increasing time constant, decreases the above effect, which means that $\frac{\theta}{\tau}$ is an important parameter in f and not θ and τ individually.

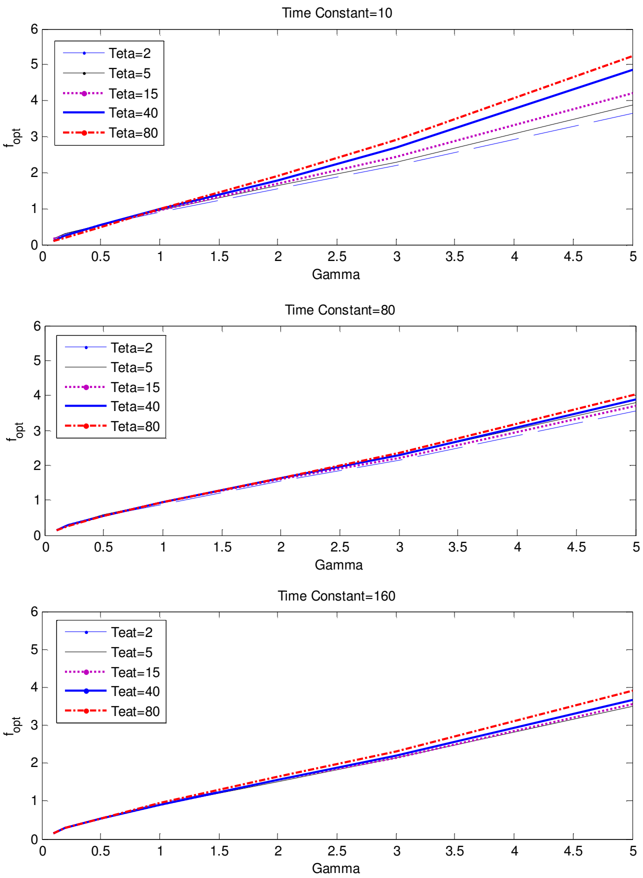

Fig. 3. Optimal f via other parameters.

These observations and some tests lead to a simple formulation for f given below

$$\lambda = fK^2\,,\;\; f = x_1\left(\frac{\theta}{\tau}+x_2\right)^{x_3}\Gamma^{x_4} \tag{7}$$

$$x_1 = 0.84\,,\;\; x_2 = 0.94\,,\;\; x_3 = 0.15\,,\;\; x_4 = 0.9$$

To show the accuracy of this formulation we compared the optimal values of f with the values obtained from (7), see Fig. 4.

This formulation removes the deficiencies of (Iglesias *et al.*, 2006) equation, for example for very small delay in comparison with time constant, this formulation yields to

$$\lambda = fK^2\,,\;\; f = 0.825\Gamma^{0.9} \tag{8}$$

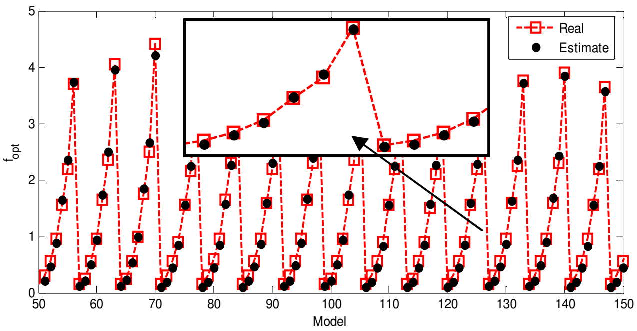

Fig. 4. Accuracy of new formulation for f.

But in this case, (Iglesias *et al.*, 2006) formulation yields to $\lambda = 0$. Also due to parameter Γ, any kind of responses can be achieved, this capability dose not exists in Cooper formulation and also smith formulation has this lag.

To have a simpler formulation, we can make parameter Γ fuzzy. We name $\Gamma = 0.1$ as output error importance, $\Gamma = 1$ as intermediate and $\Gamma = 10$ as control effort importance. With these notations, we have:

$$\lambda = \begin{cases} 0.11K^2\left(\frac{\theta}{\tau}+0.94\right)^{0.15} & \text{Output error importance} \\ 0.84K^2\left(\frac{\theta}{\tau}+0.94\right)^{0.15} & \text{Intermediate} \\ 6.67K^2\left(\frac{\theta}{\tau}+0.94\right)^{0.15} & \text{Control effort importance} \end{cases} \tag{9}$$

In special case, when the delay of system is smaller than the time constant, (10) would be simpler

$$\lambda = \begin{cases} 0.105K^2 & \text{Output error importance} \\ 0.832K^2 & \text{Intermediate} \\ 6.608K^2 & \text{Control effort importance} \end{cases} \tag{10}$$

3. SIMULATION AND EXPERIMENTAL RESULTS

3.1 Simulation Results

The first system can be found in (Shridhar and Cooper, 1997):

$$G(s) = \frac{e^{-50s}}{(150s+1)(25s+1)} \tag{11}$$

All tuning parameters for this system can be found in (Shridhar and Cooper, 1997). In this case study, tuning formulation of (Shridhar and Cooper, 1997), (Iglesias *et al.*, 2006) and proposed tuning in this paper are compared. Figure 5 shows the closed loop responses. Also Table4 summarizes the properties of different tuning methods. In Table4, IAE is integral absolute error of output, $\sum u^2$ shows the control signal energy, $\sum \Delta u^2$ shows the control effort energy, OV stands for overshoot and finally TS means settling time with criteria of 2%.

Figure 5 and also information in Table4, shows that formulation of (Shridhar and Cooper, 1997) leads to a fast response, little output error but a large control effort with a large overshoot in control signal. Tuning method of (Iglesias *et al.*, 2006) leads to a slow response in tracking and disturbance rejection and also a low control effort and a smooth control signal.

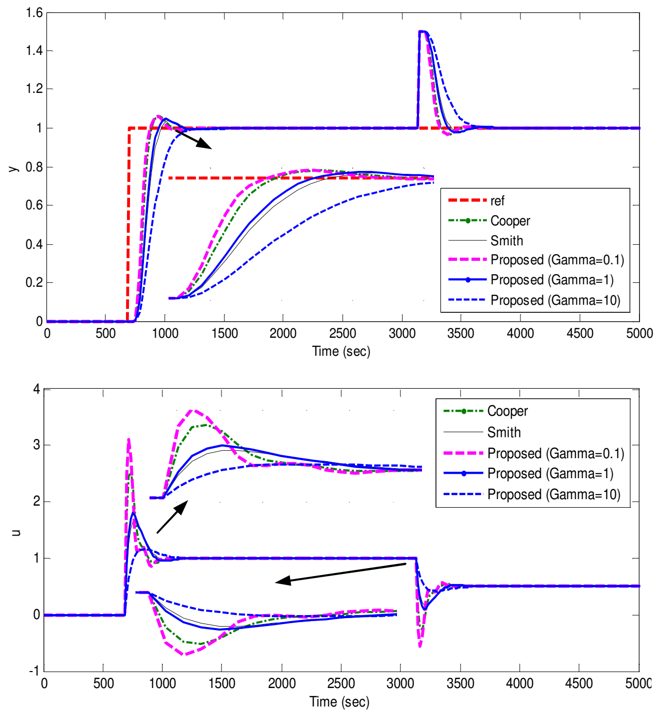

Fig. 5. Case study 1, simulation test.

Table 4. Performance comparison of DMC tuning methods, case study1.

Source	Cooper	Smith	Proposed Method		
Γ	-	-	0.1	1	10
λ	0.28	1.17	0.11	0.88	7.04
IAE	132.3	162.3	122.8	155.3	208.7
$\sum u^2$	3316	3147	3412	3177	3011
$\sum \Delta u^2$	84.98	20.51	166.3	26.8	5.89
OV in y	6	3.8	6.3	4.6	0
TS	329	411	308	394	462
OV in u	153	65	211	80	16

Finally, proposed method with $\Gamma = 0.1$ has a similar response to (Shridhar and Cooper, 1997) and $\Gamma = 10$ leads to a response that is similar to (Iglesias *et al.*, 2006). But, $\Gamma = 1$ is a good choice that has a good response in output and also a smooth control signal.

3.2 Experimental Results

In this section, the proposed algorithm is implemented experimentally. The considered plant is a pH neutralization pilot plant. This laboratory scale plant was designed and constructed in K. N. Toosi University of Technology. The structure of the plant is depicted in Fig. 6.The process consists of a continuous stirred tank reactor (CSTR) with three inputs. Acid, base and water are pumped into the CSTR by the three precise dosing pumps. A motorized mixer is used to have a well-mixed tank. Regulation of the output flow rate is done by a manual valve and this flow rate affects the time constant of the process. The pH sensor is located after the outlet valve to measure the pH of the output stream. The location of this sensor determines the transportation delay of the process. The process stream is an acetic acid (weak acid) which must be neutralized by an aquatic solution of caustic soda (strong base) as a titrating stream.

Fig. 6. pH pilot plant.

The purpose of this process is the control of the pH values in different points by base stream flow rate as a manipulated signal and to keep the pH value in neutrality in the presence of changes in acid stream flow rate as a disturbance. It is very important to pH control problem that the volume of solution in the CSTR stays in the proper constant value. So, a PI controller is used for the tank level control by the water feed rate that to be fed into the tank. The block diagram of the control system is represented in Fig. 7.

This system has a highly nonlinear behaviour, uncertainty and various kinds of disturbances (Hall and Seborg, 1989 and Nie *et. al*., 1996).

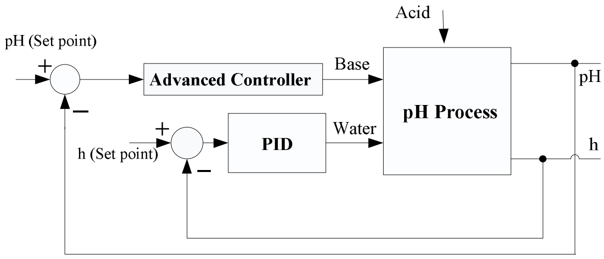

Fig. 7. Block diagram of control system.

In this case study, we consider to control the process around pH=7. The aim is both tracking and disturbance rejection in this area. A step tracking from 6.5 to 7 is required in 5620 and a disturbance rejection in 8900 is desired. This disturbance is 20% decrease in Acid flow rate. First a FOPDT model of the system is achieved by step responses as follows

$$G(s) = \frac{0.2e^{-65s}}{500s+1} \tag{13}$$

Note that to have better control performance we considered sampling time a little smaller than 50 sec. Figure 8 shows the results of DMC tuning methods. The method in (Shridhar and Cooper, 1997) yields a fast closed loop response but with high control cost. The tuning formulation of (Iglesias *et al.*, 2006) yields a slow response but a soft control signal is its advantage. Finally, the proposed method with $\Gamma = 1$ yields proper results in both output and control signal. Detailed comparison can be found in Table5.

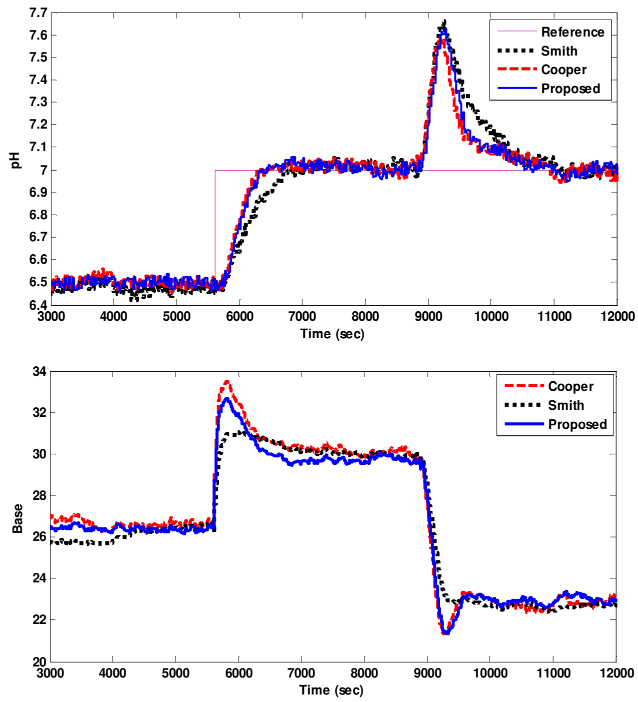

Fig. 8. Experimental test on pH pilot plant.

In Fig. 9, the level of solution in CSTR is shown, only proposed method presented. Note that in Single Input-Single

Output (SISO) pH process, the level of solution should be regulated to a fixed value. In this case study this value is 11 centimetre. This task is done by a PID controller as shown in Fig. 7.

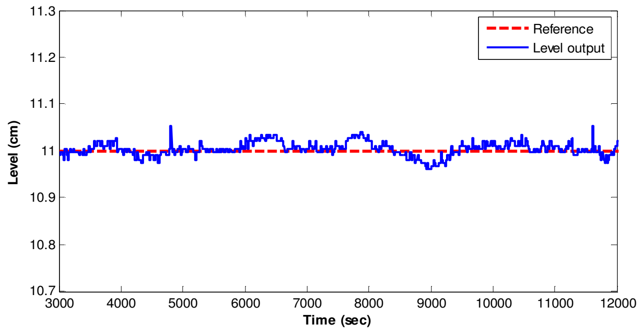

Fig. 9. Response of Level channel.

Table 5. Performance comparison of DMC tuning methods, case study2.

Source	Cooper	Smith	Proposed Method
Γ	-	-	1
λ	0.028	0.141	0.034
IAE	658.7	883.4	667.3
$\sum u^2$	2.53e+5	2.45e+5	2.49e+5
OV in y	5	4	5
TS	2400	2400	2500
OV in u	112	31	88

4. CONCLUSIONS

A new tuning method is proposed for DMC. Using FOPDT as system model, an analytically function of FOPDT parameters is obtained via ANOVA and nonlinear regression. Simulation and experimental case studies demonstrate the effectiveness of the proposed method in comparison with the two well known tuning methods.

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Appendix A. Acknowledgment

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