Novel Tuning Strategy for Two-Degree-of-Freedom PI Controllers

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Abstract: In this paper, a new tuning procedure for PI controllers in Two-Degree-of-Freedom (2DOF) structure is proposed. In design procedure, First Order plus Dead Time (FOPDT) model is used. The aim is to have good set point tracking and disturbance rejection and also maximum robustness to model uncertainties. The tuning strategy is based on using analytical rules and some conceptual rules about closed loop poles and also an exhaustive search. Simulation results demonstrate the effectiveness and validity of proposed method in coping with conflicting design objectives for a wide variety of processes including minimum phase and non-minimum phase and also integrating processes.

Keywords: 2DOF PI controller, FOPDT model, Optimization, Robustness.

1. INTRODUCTION

Proportional plus Integral plus Derivative (PID) controllers are widely used in the industry (Astrom and Hagglund, 1984; Ho *et al*., 1996). The main reason is its relatively simple structure, which can be easily understood and implemented in practice (Wang *et al*., 1999). The widespread use of PID type controllers in industry has increased efforts in the design and tuning of conventional PID controllers so as to achieve desired performance for the control system (Cheng and Hwang, 2006).

Structure of using controller is a challenging problem in control theory. Consider the typical One-Degree-of-Freedom (1DOF) structure shown in Fig. 1. Note that in a control system, the degree of freedom is defined as the number of closed-loop transfer functions that can be adjusted independently (Horowitz, 1963).

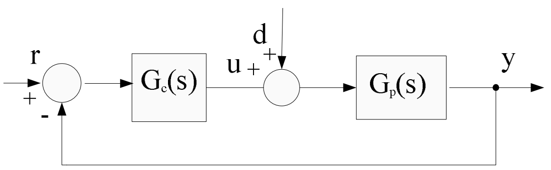

Fig. 1. Block diagram of 1DOF control structure.

In 1DOF structure, if the disturbance rejection is desired, the set-point response is often found to be poor, and vice versa. So, in some researches (Chien *et al*., 1952 and Kuwatam, 1973) on the optimal tuning of PID controllers two tables to tune controller is given, one for the “optimal disturbance rejection”, and the other one for the “optimal set point response”. The 2DOF PI controller handles such a problem, that is, in this structure both set point tracking and disturbance rejection optimization is possible. Some of advantages of using 2DOF structure are described in (Araki, 1984a). In (Araki, 1984a, b; Araki *et al*., 2003 and Yukitomo *et. al*., 2004), various 2DOF PID controllers are presented. A great number of tuning methods are presented in new researches in the structure of 2DOF. Astrom suggested a tuning approach based on Non-Convex optimization in (Astrom *et. al*., 1998). This work is one of the most powerful tuning algorithms in 2DOF structure. We will compare proposed method with this one in simulation results section. A tuning of 2DOF PID controllers within a cascade control configuration is presented in (Alfaro *et. al*., 2008). A robust tuning of 2DOF PID controllers within a cascade control configuration is presented in (Alfaro *et. al*., 2009). These two new works are a little different from proposed structure. A multi objective optimization approach is presented in (Tavakoli *et. al.,* 2007). This work will be compared with proposed method, also. Newly done work in this field is (Nemati and Bagheri, 2010). In this work a new method for tuning proposed. There are some deficiencies in this work. In this study, we try to solve these problems and present a new method. Also a comparison with this work will be done in simulation section. In this paper a new tuning formula for a 2DOF PI controller is presented. Generally, good set point response and disturbance rejection is the primary objective. And the robustness of closed loop system to modelling uncertainties is second goal.

This paper is organized as follows. In section 2, some preliminaries such as controller formulation, plant model and requirements of control is presented. In section 3 the details of design procedure is described and tuning formulations are developed. In section 4 simulation test results involved to make comparison the proposed method with other methods in some details. Finally, section 5 concludes this paper.

2. PRELIMINARIES

Now, some initial steps of design procedure are presented.

2.1 Controller and Plant Models

Block diagram of a 2DOF (or TDOF) PI control system (Astrom and Hagglund, 2000) is shown in Fig. 2.

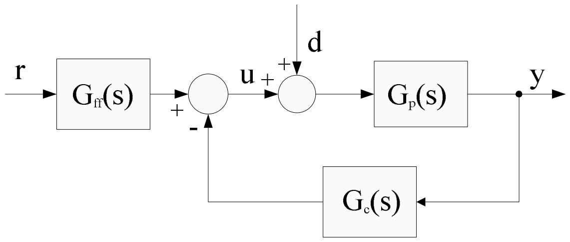

Fig. 2. Block diagram of 2DOF control system.

The process is described by transfer function $G_p(s)$. $G_c(s)$ and $G_{ff}(s)$ are respectively the feedback controller and feed forward controllers and as shown in (1) and (2) are typical PI controllers.

$$G_c(s) = k_c\left(1 + \frac{1}{T_i s}\right) \tag{1}$$

$$G_{ff}(s) = k_c\left(b + \frac{1}{T_i s}\right) \tag{2}$$

Parameter b in $G_{ff}(s)$ has no influence on disturbance rejection but it has a significant influence on the set point regulation. The tuning method is based on using First Order plus Delay Time (FOPDT) model of process. It is a common approach to approximate processes by a FOPDT transfer function if it is possible. A large number of industrial processes can be approximately modelled by it. A FOPDT transfer function is shown in (3)

$$G_p(s) = \frac{k_p e^{-\tau s}}{Ts+1} \tag{3}$$

Where k_p, T and τ arere called process gain, time constant and dead time, respectively. A simple method to obtain FOPDT model of process is based on analysis of open loop step response for stable and minimum phase processes, that is given in (Toscano, 2005) and another method for stable but non minimum phase cases is given in (Skogestad, 2003).

2.2 Control Requirements

The design objective is to tune the controller parameters to achieve three main goals:

Good disturbance rejection.

Good set point response.

Robustness to model uncertainties in FOPDT model.

Load disturbances in process control are low frequency signals added to control signal at the process input and drive the system away from its desired operating point. In this paper good load disturbance rejection is achieved through minimizing the integrated absolute error (IAE) criterion for a step signal in disturbance input.

$$IAE = \int_0^\infty |e(t)|\, dt \tag{4}$$

The second goal is to have a good set point tracking. These two goals are conflict, good design for disturbance rejection may result bad set-point tracking. Using set point weighting in 2DOF structure solves this problem.

In model based methods, the controller parameters typically are obtained from the model of process. Due to model uncertainties, the controller parameters should be tuned in a way that minimum sensitivity to these uncertainties achieves. The robustness criterion is given as:

$$s = \|S\|_\infty = max\left|\frac{1}{1+G_p(j\omega)G_c(j\omega)}\right| \tag{5}$$

$\|S\|_\infty$ is named maximum value of the sensitivity function, that is often used as a robustness measure (Astrom *et al.*, 1995) since $\|S\|_\infty$ is equal to the inverse of the minimum distance from the loop transfer function to the critical point $(-1,\ 0)$ in the Nyquist plot. Robust controller is obtained through minimizing this value.

3. DESIGNING APPROACH

In this section, the design procedure is described. The tuning procedure has two main steps. In the first step, two new design parameters are defined and we try to relate these parameters to the controller parameters. In the next step the optimal value of these two parameters are obtained

3.1 First Step

We consider two types of processes, stable processes and integrating processes.

1. Stable Processes

First, only closed loop response $(y_{CLD}(t))$ on step input disturbance $(d = 1\ and\ r = 0)$ is considered.

$$\frac{Y_{CLD}(s)}{D(s)} = \frac{G_p(s)}{1+G_c(s)G_p(s)} = \frac{k_p\left(\frac{e^{-\tau s}}{Ts+1}\right)}{1+k_p k_c\left(\frac{e^{-\tau s}}{Ts+1}\right)\left(1+\frac{1}{T_i s}\right)} \tag{6}$$

To simplify (6), we use the half rule (Skogestad, 2003) to approximate delay term in denominator

$$e^{-\tau s} \approx 1 - \tau s \tag{7}$$

By this approximation a second order polynomial for closed-loop characteristic is achieved

$$\frac{Y_{CLD}(s)}{D(s)} \approx \frac{\left(\frac{T_i}{K_c} s\, e^{-\tau s}\right)}{\left(\frac{TT_i}{k_c k_p} - \tau T_i\right)s^2 + \left(\frac{T_i}{k_c k_p} + T_i - \tau\right)s + 1} \tag{8}$$

Using a second order plus dead time model reference transfer function, with two real poles, yields to good trade off between characteristics of transient response and stability of closed-loop system.

$$\frac{Y_{CLD}(s)}{D(s)} \approx \frac{\left(\frac{T_i}{K_c} s\, e^{-\tau s}\right)}{\left(\frac{TT_i}{k_c k_p} - \tau T_i\right)s^2 + \left(\frac{T_i}{k_c k_p} + T_i - \tau\right)s + 1} = \frac{k_d s e^{-\tau s}}{(T_1 s+1)(T_2 s+1)} \tag{9}$$

Consider two new design parameters, α and T_c as

$$\alpha = \frac{T_2}{T_1} > 1,\ T_c = T_1 \tag{10}$$

According to (9), parameters k_c and T_i easily calculated as a function of parameters of plant model.

$$k_c = f_1(k_p, T, \tau, T_c, \alpha) = \frac{T[(\alpha+1)T_c+\tau]-\alpha T_c^2}{K_p((\alpha+1)\tau T_c+\tau^2+\alpha T_c^2)} \tag{11}$$

$$T_i = f_2(k_p, T, \tau, T_c, \alpha) = \frac{T[(\alpha+1)T_c+\tau]-\alpha T_c^2}{(T+\tau)} \tag{12}$$

To find the parameter b, closed loop response $\left(y_{CLR}(t)\right)$ on step input set point ($r = 1 \; and \; d = 0$) is considered

$$\frac{Y_{CLR}(s)}{R(s)} = \frac{G_{ff}(s)G_p(s)}{1+G_c(s)G_p(s)} = \frac{k_c k_p\left(b+\frac{1}{T_i s}\right)\left(\frac{e^{-\tau s}}{Ts+1}\right)}{1+k_p k_c\left(\frac{e^{-\tau s}}{Ts+1}\right)\left(1+\frac{1}{T_i s}\right)}$$
$$\approx \frac{(bT_i s+1)e^{-\tau s}}{\left(\frac{TT_i}{k_c k_p}-\tau T_i\right)s^2+\left(\frac{T_i}{k_c k_p}+T_i-\tau\right)s+1} = \frac{(bT_i s+1)e^{-\tau s}}{(T_1 s+1)(T_2 s+1)} \tag{13}$$

To obtain parameter b, it is supposed that pole-zero cancellation in (13) is occurred and the slow mode is eliminated. This pole-zero cancellation yields to fast response so the parameter b can be calculated as:

$$b = \frac{\alpha T_c}{T_i} \tag{14}$$

2. Integrating Processes

Integrating processes can be considered as a typical FOPDT with a very large constant time, T

$$G_p(s) = \lim_{T\to\infty} \frac{k_p e^{-\tau s}}{Ts+1} \cong \frac{\acute{k}_p e^{-\tau s}}{s}, \quad \acute{k}_p = \frac{k_p}{T} \tag{15}$$

Therefore, PI tuning formulas for the integrating process are obtained by using (11), (12) and (14) for a large time constant, as follows

$$K_c = \frac{(\alpha+1)T_c+\tau}{\acute{k}_p((\alpha+1)\tau T_c+\tau^2+\alpha T_c^2)}$$
$$T_i = (\alpha+1)T_c+\tau \tag{16}$$
$$b = \frac{\alpha T_c}{T_i}$$

3.2 *Second Step*

The main idea that is used in this paper is determination of parameter T_c. This parameter has direct effect on the resulting controller so it should be chosen such that the closed loop performance is satisfied and the designed controller is physically realizable (Chen *et al.*, 2003). To choose parameter T_c, some important notes should be considered:

Remark 1- T_c is related to the open loop time constant and delay time. It is true to say that in stable systems T_c also should be function of $\frac{\tau}{T}$. And in integrating systems, T_c should be function of delay time τ.

Remark 2-Systems with large delay time should not be expected to have so fast closed loop response in comparison with open loop response. So in this case T_c should be a little large.

Remark 3-For systems with a small value of $\frac{\tau}{T}$, it is possible to choose T_c small to have fast response.

Remark 4-Decreasing T_c leads to a fast closed loop response.

In previous work (Nemati and Bagheri, 2010), T_c is chosen as:

$$T_c = \sqrt{\frac{T\tau}{T+\tau}} \tag{17}$$

This choice exhibits that for processes with large time delay, the time constant of closed loop system is smaller than time constant of open loop system, so the closed loop response is fast and also if this value decreases to bellow 1, then second root of it become large than open loop value and it is not desirable.

$$\tau \gg T \implies T_c \to \sqrt{T} \tag{18}$$

Also, for small value of τ, time constant of closed loop system tend to zero and unstable closed loop appears.

$$\tau \to 0 \implies T_c \to 0 \tag{19}$$

In this paper, according to remark1-4, desired time constant considered as:

$$T_c = \frac{\frac{T+\frac{\tau}{10}}{1.5}+\frac{T}{7}e^{-5\left(\frac{\tau}{T}-1\right)}}{1+e^{-5\left(\frac{\tau}{T}-1\right)}} \tag{20}$$

Figure 3 shows a plot of this equation, in this figure we have $T = 10$. In this formulation, for a system with small time delay, closed loop system will be up to 7 times faster than open loop. For a system with large time delay the closed loop time constant will be a large enough value.

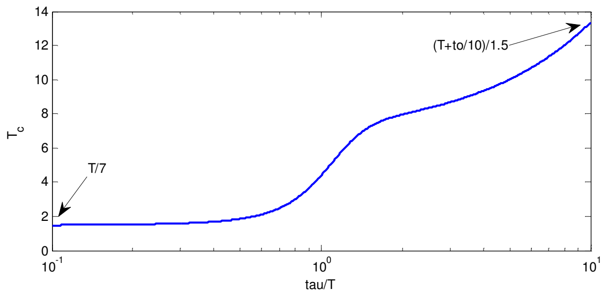

Fig. 3. The choice of closed loop time constant according to open loop information (stable systems).

For integrating processes a new choice for T_c is as:

$$T_c = \frac{\left(1+\frac{\tau}{2}\right)+0.2e^{-5(\tau-1)}}{1+e^{-5(\tau-1)}} \tag{21}$$

Figure 4 shows closed loop time constant of integrating processes according to (21). In this formulation, when delay of system is small then a fast response due to small T_c is achievable and for large time delay, an enough large value of T_c is obtained.

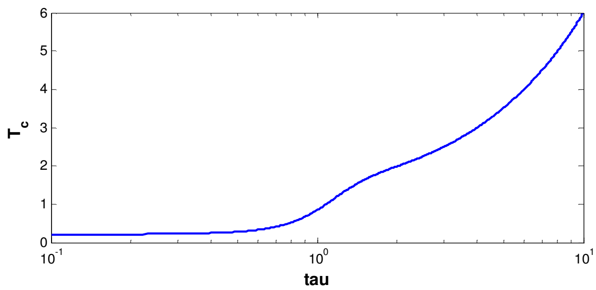

Fig. 4. The choice of closed loop time constant according to open loop information (integrating systems).

Finally now we should determine the optimal value of α. This parameter should be determined so that the objective function $\|S\|_\infty$ and IAE both in disturbance rejection and set point regulation is acceptable. Using of exhaustive search for a wide range of different FOPDT models, optimal value of α is obtained as:

$$\alpha = 3 \tag{22}$$

4. SIMULATION

In this section performance of proposed method is compared with two well known methods and a newly done work through three simulation examples. These methods are presented in (Astrom *et al.*, 1998) and (Tavakoli *et al.*, 2007). These methods are known as APH and MO respectively. Newly done work is (Nemati and Bagheri, 2010).

The scenario of comparison consists of three parts: set point tracking, disturbance rejection and robustness to model uncertainties. For testing tracking performance, a unit step changes is applied in the beginning of simulation, also for disturbance rejection capability test, in middle of simulation an input step disturbance is entered to system and finally to cope with uncertainties, in the last part of simulation, 20% changes in DC gain of plant is considered.

4.1 Example 1: Forth Order System

Consider a system with transfer function

$$G_{p1}(s) = \frac{1}{(s+1)(0.2s+1)(0.04s+1)(0.008s+1)} \tag{23}$$

$G_{p1}(s)$ is a easy to control system with wide range of real poles. In order to tune PI parameters by proposed methods, a FOPDT model of this system is required. A simple method based on open loop step response is introduced in (Toscano, 2005). Using this approximation method yields to:

$$G_{m1}(s) = \frac{e^{-0.224s}}{1.084s+1} \tag{24}$$

The closed loop step response, disturbance rejection and robustness test of new proposed, previous proposed, MO and APH methods are shown in Fig. 5. Note that we named previous proposed for (Nemati and Bagheri, 2010), APH for (Astrom *et al.*, 1998) and MO for (Tavakoli *et al.*, 2007).

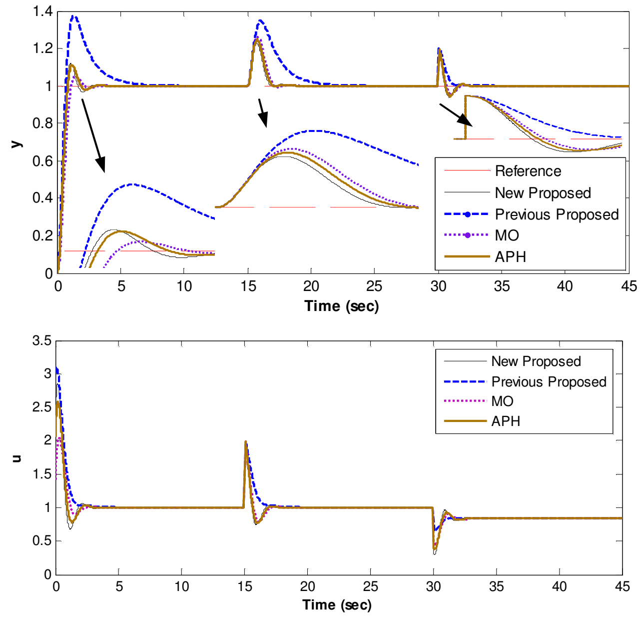

Fig. 5. Closed loop responses of $G_{p1}(s)$ using previous proposed, new proposed and MO methods

A detailed comparison can be found in Table1. In this table, IAE shows the integral absolute error, TS stands for settling time with criteria of 2%, OV is overshoot and S, D, R are set point tracking, disturbance rejection and robustness case, respectively.

According to Fig. 5 and table 1, for $G_{p1}(s)$ we can say newly proposed method in tracking and disturbance rejection has the best response, also robustness of this method is more than others.

4.2 Example 2: Non-Minimum Phase System

To investigate the performance of the proposed tuning method in coping with non-minimum phase processes, the transfer function $G_{p2}(s)$ is considered

$$G_{p2}(s) = \frac{-2s+1}{(s+1)^3} \tag{25}$$

Using the half rule method, introduced in (Skogestad, 2003), the FOPDT model for this non-minimum phase system can be calculated as:

$$G_{m2}(s) = \frac{e^{-3.5s}}{1.5s+1} \tag{26}$$

Fig. 6 shows the results of simulation. Again detailed performance comparison is made in Table 1. In tracking after MO, newly proposed method has better performance. In disturbance rejection this method has poor performance in comparison with other methods. Finally robustness of newly proposed method after MO is more than other methods.

4.3 Example 3: Integrator with Large Delay

$$G_{p3}(s) = \frac{e^{-5s}}{s} \tag{27}$$

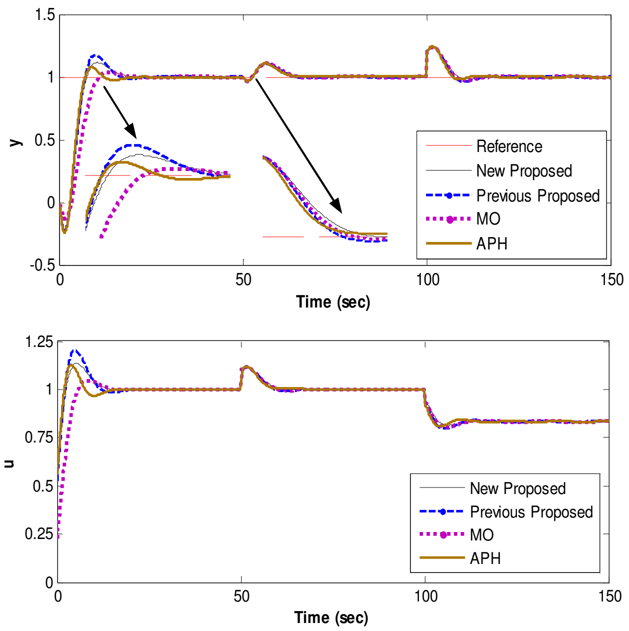

Fig. 6. Closed loop responses of $G_{p2}(s)$.

$G_{p3}(s)$ is a pure integrator with a large time delay, that is a difficult system to control. The results of simulation can be found in Fig. 7. Refer to Table 1 for detailed information of the simulation. In this simulation newly proposed method in tracking and disturbance rejection has the best performance but robustness of MO is more than it. As it mentioned previously proposed method in dealing with high delay time systems shows poor performance. This fact can be shown in Fig. 7.

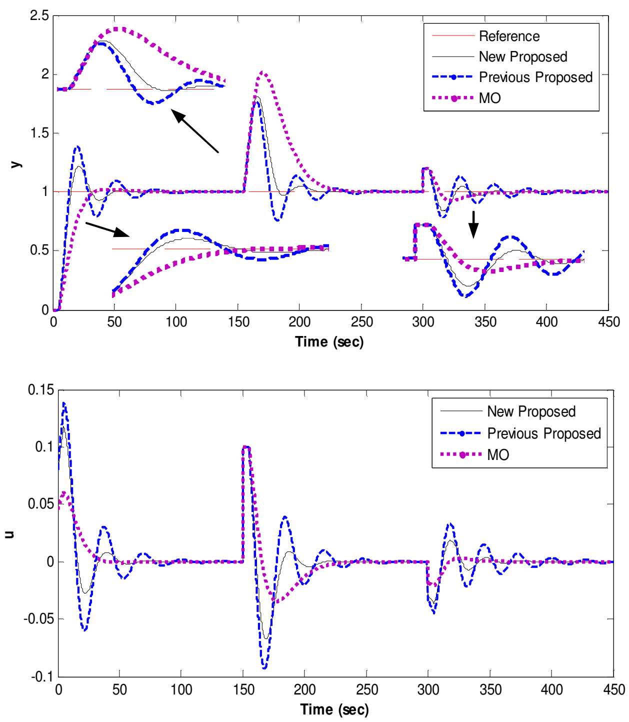

Fig. 7. Closed loop responses of $G_{p3}(s)$.

TABLE1. PERFORMANCE COMPARISON OF THE METHODS.

Plant	Method	New	Previous	MO	APH
$G_{p1}(s)$	K_c	3.128	1.565	2.4	2.74
	T_i	0.672	1.188	0.66	0.67
	b	0.74	1.924	0.59	0.75
	IAE_S	0.52	1.14	0.62	0.55
	TS_S(sec)	2.36	5.4	1.92	2.4
	OV_S(%)	12.1	37.5	5.5	11.1
	IAE_D	0.215	0.76	0.28	0.25
	TS_D(sec)	1.62	5.1	1.9	1.7
	OV_D(%)	23	34.6	26.5	24.5
	IAE_R	0.217	0.254	0.231	0.222
	TS_R(sec)	1.3	1.3	1.4	1.4
	OV_R(%)	20	20	20	20
$G_{p2}(s)$	K_c	0.237	0.277	0.26	0.36
	T_i	1.618	1.65	1.64	2.3
	b	2.285	1.9	0.87	1.54
	IAE_S	6.14	6.37	6.94	5.62
	TS_S(sec)	16.9	15.9	16.9	16.5
	OV_S(%)	11.3	17.3	3.6	7.5
	IAE_D	8.07	7.68	7.8	7.5
	TS_D(sec)	16.9	21.4	16.5	18
	OV_D(%)	103	106	104	106
	IAE_R	2.70	2.77	2.72	2.4
	TS_R(sec)	32	33.3	33.2	29.2
	OV_R(%)	23.1	23.7	23.5	24
$G_{p3}(s)$	K_c	0.144	0.16	0.091	-
	T_i	19	13.94	29.17	-
	b	0.55	0.48	0.5	-
	IAE_S	13.24	17.26	15.86	-
	TS_S(sec)	46.7	86	51.5	-
	OV_S(%)	22	40	2.2	-
	IAE_D	13.6	14.6	32	-
	TS_D(sec)	60.5	84.5	32.08	-
	OV_D(%)	82.5	77	78	-
	IAE_R	8.10	14.41	7.41	-
	TS_R(sec)	50.5	100	44	-
	OV_R(%)	20	20	20	-

5. CONCLUSIONS

A new tuning approach for 2DOF PI controller based on FOPDT model of process is proposed in this study. The design problem considers three essential requirements of control problem. These requirements are load disturbance rejection, set point regulation and robustness to model uncertainties. The main advantage of proposed method is its simplicity and analytically equations for controller parameters. The simulation results show that, over all, the proposed tuning method can effectively satisfy conflicting design requirements and also results show that this method has better performance in comparison with other well-known methods.

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